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MEASURING THE IMPACT OF VULNERABILITY ON THE NUMBER OF POOR: A NEW METHODOLOGY WITH EMPIRICAL ILLUSTRATIONS

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Abstract

Given a poverty line, a person who is non-poor (poor) currently may not be treated as non-poor (poor) in a vulnerable situation. This paper looks at the impact of vulnerability on the poverty line. The poverty line is adjusted in the presence of vulnerability such that the utility of a person at the current poverty line and that at the adjusted poverty line become identical. Using an additive model of vulnerability, it is shown that if the utility function obeys constant Arrow-Pratt absolute risk aversion, then the harmonized poverty line is a simple absolute augmentation of the current poverty line. On the other hand, under a multiplicative model of vulnerability with constant Arrow-Pratt relative risk aversion, the revised poverty line is a simple relative augmentation of the current poverty line. The paper contains empirical illustrations which assume that constant relative risk aversion applies to countries involved in the Asia-Pacific region. Upward adjustment of the poverty line under increased vulnerability, as captured through the value of the risk aversion parameter, is also observed.

JEL Classification: D84, I32

Contents

1. INTRODUCTION

In the dimensions of income and health, vulnerability is the risk that a household or an individual will experience an episode of income or health poverty over time. But vulnerability also means the probability of being exposed to a number of other risks (violence, crime, natural disasters, having to leave school) (World Bank 2000: 19). The focus of vulnerability should hence be on the risk of negative outcomes in the future (Hoddinott and Quisumbing 2003), where most generally 'negativity' refers to a situation in which an individual is below the poverty line (Calvo and Dercon 2013). Vulnerability, thus, imposes a security risk on individuals in the sense that it affects their well-being negatively. It may cause long-term deprivation for individuals.

"The challenge of development includes not only the elimination of persistent and endemic deprivation, but also the removal of vulnerability to sudden and severe destitution" (Sen 1999: 1). "Protecting vulnerable groups during episodes of macroeconomic contraction is vital to poverty reductions in developing countries" (World Bank 1997: 1).

In the measurement of vulnerability, we need to be concerned, not only with current conditions, such as current income and consumption, but also with the risks an individual faces and their ability to avoid, reduce and overcome these. This shows that an indicator of vulnerability should take several appropriate factors into account. For concreteness, in the remainder of this paper we assume that the unit of analysis is an individual and income represents the underlying economic variable.

As Klasen and Povel (2013) pointed out, vulnerability at the household/individual level can be broadly subdivided into the following categories: (i) vulnerability as uninsured exposure to risk; (ii) vulnerability as low expected utility; (iii) vulnerability as expected poverty; and (iv) vulnerability to poverty. The first three categorizations of vulnerability were analyzed, among others, by Hoddinott and Quisumbing (2003), Ligon and Schechter (2004) and Gaiha and Imai (2009) (see also Hoogeveen, et al. 2004). Vulnerability to poverty was introduced and discussed by Calvo and Dercon (2013). (See Fujii (2013) for a recent discussion.)

Vulnerability, as uninsured exposure to risk, indicates whether income shocks render changes in consumption (see Townsend 1994; Amin, Rai and Topa 2003; and Skoufias and Quisumbing 2003). This notion of vulnerability is concerned with changes in the current level of consumption not with the levels of consumption. It does not take into account an individual's attitudes towards risks.

Vulnerability, as low expected utility, relates vulnerability with variability. There is a long history of the use of the variance as a measure of risk in statistical decision theory (Rothschild and Stiglitz 1970). It was rigorously formulated by Ligon and Schechter (2003). The Ligon and Schechter notion of vulnerability is measured by the difference between the utility derived from a threshold income and the individual's expected utility derived from incomes in a vulnerable situation. The higher is the difference between the two utility values, the more vulnerable the person is. The individual is non-vulnerable in this situation if his income is above the threshold limit (see also Glewwe and Hall 1998; Dercon 2002; and Coudouel and Hentschel 2000). The major advantage of this approach is that it incorporates an individual's attitudes towards risks explicitly by making the formulation directly dependent on the von Neumann-Morgenstern utility function. In view of the non-constancy of the utility function and probabilistic formulation, the approach takes into account the severity and likelihood of shocks on individual welfare. One limitation of this approach is that all individuals are assumed to possess the same attitudes towards risks. It is, however, true that under non-comparability of individual utility functions, aggregation is not possible under usual Arrowian axioms (Sen 1977; Boadway and Bruce 1984; and Blackorby, Donaldson and Weymark 1984). The Ligon-Schechter framework was also assumed by Elbers and Gunning (2003) by incorporating explicitly the future streams of income over an infinite time horizon.

Vulnerability, as expected poverty, refers to the risk of an individual's income falling below the poverty line. The idea was initiated by Ravallion (1988) and advanced and analyzed further by Holzmann and Jorgensen (1999). A formal analysis of this approach was developed by Chaudhuri, Jalan and Suryahadi (2002), which indicates the probability that an individual's income will be below an exogenously-given poverty line. However, it does not take into account the sensitivity towards risks. An individual's position with respect to vulnerability is simply decided in terms of some expected income. Hoddinott and Quisumbing (2003) address this shortcoming by expressing vulnerability as expected poverty using the Foster-Greer-Thorbecke (1984) poverty index. Interpreting the negative of poverty as utility, we note that the Arrow-Pratt absolute risk-aversion measure for this utility function increases as the value of the underlying parameter increases. However, such a risk preference is not unambiguously supported by empirical findings (see Hoddinott and Quisumbing 2003; and Binswanger 1981). Empirical applications of this approach can be found in Hoddinott and Quisumbing (2003), Surayahdi and Sumarto (2003), Kamanou and Morduch (2004), Christiaensen and Subbarao (2004) and Gunther and Harttgen (2009).

The notion of vulnerability to poverty was introduced by Calvo and Dercon (2013). Instead of starting from individual poverty or a utility function, they developed an axiomatic characterization of a vulnerability measure. In this framework, vulnerability is a weighted average of future state-wise deprivations, where the weights are the probabilities of outcomes associated with different states of the world in the future. The two measures that were characterized by Calvo and Dercon (2013) are the expected measures of Chakravarty (1983) and Watts (1967). These measures are, in fact, expected poverty measures. [1] These measures explicitly take into account risk aversion. They rely on the poverty line, assigned probabilities and relevant states of the world. Earlier, Dutta, Foster and Mishra (2011) axiomatically derived a vulnerability measure, which unlike the Calvo-Dercon measure, assumes that deprivation depends explicitly on the current and future incomes. Therefore, this measure allows us to look at relative changes under vulnerability.

The objective of this paper is to study the implications of vulnerability on the poverty line. More precisely, we investigate the issue of adjusting the poverty threshold under vulnerability so that the corrected poverty line also represents the subsistence standard of living in an environment of vulnerability. Essential to the adjustment is the assumption that the utility derived from the existing poverty line is the same as the expected utility generated by the new poverty line affected by a random error (noise) representing vulnerability. Thus, the formulation relies on the implicit assumption that vulnerability is treated as low expected utility. Under certain realistic assumptions about the noise, in an additive model the improved poverty line is shown to exceed the existing poverty line by a constant amount if the utility function displays constant Arrow-Pratt absolute risk aversion (see Arrow 1965; and Pratt 1964). Likewise, in a multiplicative model, the adjusted poverty line becomes a scale transformation, where

[1] Chakravarty, Chattopadhyay and Qingbin (2015) explored a partial ordering of vulnerability to poverty induced by expected poverty indices. See also Hardeweg, Wagener and Waibel (2013) for a stochastic dominance-based partial ordering.

the underlying scalar is greater than unity, if the utility function exhibits constant Arrow-Pratt relative risk aversion.

In a recent contribution, Dang and Lanjouw (2014) suggested two formal approaches to setting the vulnerability line. In the first approach, they identified a subgroup of a population which is clearly not vulnerable and defined the vulnerability line as the lower-bound income for this population subgroup. The second approach considers a subgroup which is not poor but faces a real risk of falling into poverty. They set the upper-bound income for this subgroup as the vulnerability line. While our approach relies on the Arrow-Pratt theory of risk aversion, the Dang-Lanjouw approach is based on a probabilistic formulation. Therefore, neither supplements the other; the two approaches are clearly different.

This paper is organized as follows. The next section presents a brief overview of the background material involving the Arrow-Pratt measures of risk aversion. Section 3 formally presents the derivation of the vulnerability-adjusted poverty lines under alternative assumptions about the Arrow-Pratt measures. The focus of Section 4 is on the estimation of the variance of the noise which characterizes the uncertain income, whereas Section 5 presents an empirical illustration using data from the Asian-Pacific region. Section 6 finally concludes.

2. BACKGROUND

It is often useful to have an indicator of risk aversion. A risk-aversion indicator is a measure of the extent to which an individual becomes averse to risky situations. It is helpful to make a comparison between two individuals in terms of their attitudes towards risk.

Let $U:(0,\infty)\rightarrow\Re$ denote the utility function of the individual under consideration, where $\Re$ denotes the real line. The utility function U is assumed to be continuous, increasing and strictly concave. For our purposes, we assume also that it is at least twice differentiable. We denote the first and second derivatives of U by U' and U'' respectively. Since it is assumed that U is a monotone increasing and strictly concave function, $U'>0$ and $U''<0$ are satisfied.

The Arrow-Pratt measure of absolute risk aversion $AP_A(M)$, for a person with utility function U and level of income M, is defined as:

$$AP_A(M)=-\frac{U''(M)}{U'(M)}. \quad (1)$$

The indicator $AP_A(M)$ takes on a positive, zero or negative value depending on whether an individual is risk-averse, risk-neutral or risk-preferring, i.e., the utility function is strictly concave, affine or strictly convex. A higher value of AP_A indicates that an individual's aversion towards risk is higher.

If incomes are expressed in relative terms, an appropriate measure that indicates attitudes to risk is the Arrow-Pratt relative risk aversion measure defined as:

$$AP_R(M)=-\frac{M\times U''(M)}{U'(M)}. \quad (2)$$

The measure AP_R takes on positive, zero or negative values depending on whether an individual is risk-averse, risk-neutral or risk-preferring.

Highly significant implications of constant and strictly monotonic risk-aversion measures that can motivate a focus on these concepts arise in the context of analysis of the cost of risk and portfolio formation. One way of looking at the cost of risk or the risk premium is to define it as the difference between the expected income on a risky prospect and the certainty equivalent, the certain amount of income that is equally preferred to the prospect. It shows how much the individual would be willing to pay rather than face the risky prospect (see Gravelle and Rees 2012). Formally, it is defined as:

$$C_A\left(\underline{p},\underline{x}\right)=\sum_{i=1}^{k} p_i x_i - x_e\ , \tag{3}$$

where $\underline{x}=\left(x_1,x_2,...,x_k\right)$ is the vector of state-contingent returns on the risky prospect; k is the number of states; p_i is the probability of state i; and $\underline{p}=\left(p_1,p_2,...,p_k\right)$. Now $\sum_{i=1}^{k} p_i x_i$ is the expected return and the certainty equivalent x_e is implicitly defined by $\sum_{i=1}^{k} p_i U\left(x_e\right)=\sum_{i=1}^{k} p_i U\left(x_i\right)$. The indicator C_A is a cost of risk because, in the absence of uncertainty, it is zero and it is positive for a risk-averse person if the environment is characterized by uncertainty. This cost remains invariant under equal absolute changes in outcomes on the prospect if and only if AP_A of the underlying utility function is a constant (Chakravarty 2013).

Likewise, the relative cost can be defined as the proportionate gap between the expected return on the prospect and the certainty equivalent. Formally, it is given by:

$$C_R\left(\underline{p},\underline{x}\right)=1-\frac{x_e}{\sum_{i=1}^{k} p_i x_i}. \tag{4}$$

If uncertainty prevails, this cost is positive under strict concavity of the utility function. The cost measure C_R remains invariant when the scale of outcomes changes by a positive scalar if and only if AP_R of the underlying utility function is a constant (Chakravarty 2013).

In a portfolio consisting of one risky asset and one risk-free asset, the amount invested in the risky prospect increases with an increase in his/her wealth if the absolute risk-aversion measure is decreasing. That is, as a person becomes less risk averse with an increase in the level of wealth, his/her demand for the risky asset increases. This means that the risky prospect is a normal good (Arrow 1970). Likewise, if the relative risk aversion measure is increasing, then the share of wealth invested in the risky prospect decreases with an increase in the level of wealth (see Demange and Laroque 2006).

3. FORMAL FRAMEWORK

In this section, we investigate the impact of vulnerability on the poverty line. A person at the poverty line z_0 without vulnerability has a certain utility $U(z_0)$. On the other hand, in a vulnerable situation, he/she is subjected to an uncertain income. We deal with the cases of constant and relative risk aversion in the next two subsections, respectively.

3.1 Constant Absolute Risk Aversion

In this subsection, we assume that the individual's income is characterized by an additive noise ε, a random variable whose mean is 0 and variance is σ^2. Such an assumption for the error process in consumption was made by Ligon and Schechter (2003). Rothschild and Stiglitz (1970) assumed this type of additive noise in their well-known study on defining increasing risk. In our case, the noise term represents vulnerability. Hence, the individual's income is now $z_1 + \varepsilon$ and the corresponding state-dependent utility is $U(z_1 + \varepsilon)$, where z_1 is the new poverty line. We refer to this formulation as the additive noise model.

We assume a utility consistency condition, which says that utility derived from the given poverty line z_0 and the expected utility from the poverty line z_1, accompanied by the noise, should be equal. That is, the poverty line z_1 should be such that the person becomes indifferent between the expected utility from the vulnerable income $z_1 + \varepsilon$ and the certain utility from z_0. Thus, $U(z_0) = E(U(z_1 + \varepsilon))$, where *E* stands for the expectation operator. This idea is similar in spirit to the notion of certainty equivalent and risk-neutral valuation employed in the theory of finance. According to risk-neutral valuation, the current period stock price is the discounted present value of the expected value of the future period stock prices, where the discounting is done using a risk-free rate of interest (Demange and Laroque 2006).

Expanding the right-hand side of the expression $U(z_0) = E(U(z_1 + \varepsilon))$ by Taylor's expansion around z_1, we have $U(z_0) = E\left(U(z_1) + \varepsilon U'(z_1) + \left(\frac{\varepsilon^2}{2}\right)U''(z_1) + \ldots\right)$. Ignoring higher-order terms greater than 2, we have:

$$U(z_0) = U(z_1) + \left(\frac{\sigma^2}{2}\right)U''(z_1). \tag{5}$$

Given that $U'' < 0$, we obtain, $U(z_0) - U(z_1) = \left(\frac{\sigma^2}{2}\right)U''(z_1) < 0$, which implies that $z_1 > z_0$.

Intuitively, this is a quite reasonable result. Because z_1 is the poverty line in the presence of vulnerability, the value of z_1 should be higher that of z_0 so that with the additional income the individual can cope with the disturbance in income generated by vulnerability and becomes equally well off as he was with z_0.

We can rewrite equation (5) as $F(z_0, z_1) = 0$, where F is a real-valued function defined on the positive part of the two-dimensional Euclidean space. By the implicit function theorem, we can solve $F(z_0, z_1) = 0$ for z_1 as a function of z_0 (Apostol 1971).

We are interested in finding non-trivial solutions and we try some special cases for which non-trivial solutions can be found by inspection. As a simple trial, suppose $U(.)$ satisfies constant absolute risk aversion. That is, $U(z)=A-Be^{-\alpha z}$, where $\alpha>0$, $B>0$ and A are constants. In fact, α is the constant value of the absolute risk aversion measure.

The first derivative is written as $U'(z) = -B(-\alpha)e^{-\alpha z} = B\alpha e^{-\alpha z}$.

The second derivative is expressed as $U''(z) = \alpha B(-\alpha)e^{-\alpha z} = -B\alpha^2 e^{-\alpha z}$.

Thus, equation (5) implies that:

$$A - Be^{-\alpha z_0} = A - Be^{-\alpha z_1} + \left(\frac{\sigma_A^2}{2}\right)(-B\alpha^2)e^{-\alpha z_1}$$

where σ_A^2 denotes the variance in this absolute case.

This leads to the result:

$$z_1 = z_0 + \left(\frac{1}{\alpha}\right) ln\left(1 + \left(\frac{\sigma_A^2}{2}\right)\alpha^2\right). \tag{6}$$

Thus the adjusted poverty line z_1 is easily estimated[2] on the basis of:

- the original poverty line, z_0;
- the variance σ_A^2 of the error ε; and
- the coefficient of absolute risk aversion, α.

From equation (6), it follows that $z_1 = z_0 + \beta$, where $\beta > 0$ is a constant, depending only on α and σ. That is, z_1 is a positive translation of z_0. Thus, with a constant absolute aversion to risk we get that the new poverty line is an absolute positive shift of the existing poverty line. The term, β, may be regarded as a compensation factor for vulnerability. For instance, for a poor country where the poverty line is assumed to cover only basic needs, if one wants to take vulnerability into account, one just moves upward the original poverty line (the one that ignores vulnerability) by a constant. This absolute shift does not depend on the existing poverty line. It is explicitly dependent on the noise representing vulnerability and the nature of risk aversion given by the utility function.

3.2 Constant Relative Risk Aversion

Assume again that a person at the poverty line z_0 without vulnerability has a certain utility $U(z_0)$. Assume now that the individual is subjected to an uncertain income characterized by the proportional noise ε defined above. The individual's income is now $z_2(1+\varepsilon)$ and the corresponding state-dependent utility is $U\big(z_2(1+\varepsilon)\big)$. Given that there is indifference in the two situations, $U(z_0) = E[U\big(z_2(1+\varepsilon)\big)]$. Expanding the right-hand side by a Taylor's expansion, we have:

$$U(z_0) = E\left\{U(z_2) + \varepsilon z_2 U'(z_2) + \frac{\varepsilon^2}{2} z_2^2 U''(z_2) + \cdots\right\}.$$

[2] Using the definition of equation (1), it is easily shown that $AP_A(z) = \alpha$ for this model.

Taking approximations, we obtain:

$$U(z_0) = U(z_2) + \frac{\sigma_R^2}{2} z_2^2 U''(z_2) \tag{7}$$

where σ_R^2 is now the variance of ε.

We thus have $U(z_0) - U(z_2) = \frac{\sigma_R^2}{2} z_2^2 U''(z_2) < 0$ because $U'' < 0$. Hence, $z_2 > z_0$. Again, in view of the implicit function theorem, we can always solve z_2 in terms of z_0.

Let the utility function be defined as $U(z) = A_1 + B_1 \frac{z^{1-\delta}}{1-\delta}$, where $B_1 > 0$, A_1 and $0 < \delta \neq 1$ are constants.

The first and second derivatives are:

$$U'(z) = B_1 \frac{1}{1-\delta}(1-\delta)z^{-\delta} = B_1 z^{-\delta}; \text{ and } U''(z) = B_1(-\delta)z^{-\delta-1}.$$

On the basis of the utility function previously defined, using equation (7), we obtain:

$$A_1 + B_1 \frac{(z_0)^{1-\delta}}{1-\delta} = A_1 + B_1 \frac{(z_2)^{1-\delta}}{1-\delta} + \frac{\sigma_R^2}{2}(z_2)^2 B_1(-\delta)(z_2)^{-\delta-1}.$$

Again, by simple but tedious algebra, we obtain:

$$z_2 = z_0 \left[1 - \delta(1-\delta)\frac{\sigma_R^2}{2}\right]^{-1/(1-\delta)}. \tag{8}$$

Here also it is easy to calculate[3] the adjusted poverty line z_2 on the basis of:

- the original poverty line z_0;
- the variance σ_R^2; and
- the coefficient of relative risk aversion, δ.

We now consider the particular case where $\delta = 1$. We may then write that:

$$U(z) = A_1 + B_1 ln z.$$

Thus we obtain

$$U'(z) = B_1\left(\frac{1}{z}\right) \text{ and } U''(z) = -B_1 \frac{1}{z^2}.$$

Again, using equation (7), we derive that:

$$A_1 + B_1 ln z_0 = A_1 + B_1 ln z_2 + \frac{\sigma_R^2}{2}(z_2)^2(-B_1)\frac{1}{(z_2)^2}$$

[3] It may be noted that, using equation (2), we obtain $AP_R(q) = \delta$ for this proportional model.

which leads to:

$$lnz_2 = lnz_0 + \frac{\sigma_R^2}{2} \text{ and } z_2 = z_0\beta = z_0 e^{(\sigma_R^2/2)} \quad (9)$$

where β is defined as $ln\beta = \frac{\sigma_R^2}{2}$.

To compute z_2, we need only to know σ_R^2 and the original poverty line z_0.

From equation (9), it is clear that the constant relative risk aversion utility is not consistent with an additive shift of the poverty line.

In short, we note that the proportionally adjusted poverty line can be justified by assuming a constant relative risk aversion utility under a multiplicative model of vulnerability to poverty. For instance, for a country where the poverty line is also assumed to take into account the "cost of social inclusion" (relative poverty), if one wants to take vulnerability into account one would have to implement a scale transformation of the original poverty line (the one ignoring vulnerability).

We may now summarize the previous observations in the following two propositions.

Proposition1: In the additive noise model, under constant absolute risk aversion, the vulnerability-adjusted poverty line is a positive translation of the existing poverty line. In this additive model, the translation shift is not supported by a constant relative risk aversion utility function.

Proposition2: In the multiplicative noise model, under constant relative risk aversion, the vulnerability-adjusted poverty line is a relatively augmented transformation of the existing poverty line. In this multiplicative model, the scale transformation is not supported by a constant absolute risk aversion utility function.

4. ESTIMATION OF THE VARIANCE IN THE MULTIPLICATIVE CASE[4]

Given some income distribution which is supposed to be subject to vulnerability, we need to derive the variance $V(\varepsilon)$ (denoted above by σ_R^2) in the multiplicative case.

Let *X* denote income that would be observed if there was no vulnerability and let z_0 be the poverty line in such a case at, say, time 0. Assume that, at some time *t,* the appropriate income variable is Y_t but this is assumed to be subject to vulnerability, in the sense that it is generated by taking into account the presence of a noise term ε_t in addition to the existing distribution at time 0. We assume a multiplicative model:

$$Y_t = X(1 + \varepsilon_t) \quad (10)$$

where X and ε_t are assumed to be uncorrelated.

[4] As stressed below at the beginning of Section 5, in the additive case, the parameter value depends on the unit of measurement of the income, consumption or other well-being variable, so that implementing such an approach becomes very difficult. Hence, we concentrate our attention on the multiplicative case.

Hence

$$\ln Y_t = \ln X + \ln(1+\varepsilon_t) \quad (11)$$

Assume we have information on *T* distributions, Y_t, *t=1,2,…, T.*

We therefore write:

$$\left(\frac{1}{T}\right)\sum_{t=1}^{T} lnY_t = lnX + \left(\frac{1}{T}\right)\sum_{t=1}^{T} ln\,(1+\varepsilon_t).$$

Under first-order approximation of $ln(1+\varepsilon_t)$ we may write that $ln(1+\varepsilon_t) \approx \varepsilon_t$ so that

$$\left(\frac{1}{T}\right)\sum_{t=1}^{T} lnY_t = lnX + \left(\frac{1}{T}\right)\sum_{t=1}^{T} \varepsilon_t. \quad (12)$$

We rewrite equation (12) as

$$\left(\frac{1}{T}\right)\sum_{t=1}^{T} lnY_t \approx lnX + \bar{\varepsilon}$$

where $\bar{\varepsilon}$ is the average of all the ε_t. Assume that the variance σ^2_{Rt} of ε_t is the same for all *t,* so that we can write that

$$\sigma^2_{Rt} = \sigma^2_R .$$

The variance $Var(\bar{\varepsilon})$ of $\bar{\varepsilon}$ will then be expressed as

$$Var(\bar{\varepsilon}) = \frac{\sigma^2_R}{T},$$

so that $Var(\bar{\varepsilon}) \to 0$ if σ is small or if $T \to \infty$.

It then follows that

$$\left(\frac{1}{T}\right)\sum_{t=1}^{T} lnY_t \approx lnX. \quad (13)$$

From equation (13), we obtain

$$V(lnX) \cong V\left[\left(\frac{1}{T}\right)\sum_{t=1}^{T} lnY_t\right] = \left(\frac{1}{T^2}\right)\sum_{t=1}^{T} V(lnY_t). \quad (14)$$

We assume that we have data, for each period *t*, and that the observations on Y_t have as typical element, an income, Y_{it}, with *i* varying from 1 to *n* (e.g., *n* = 100,000). We, therefore, also have, for each period *t,* observations on lnY_t whose typical element is lnY_{it}. As a consequence, we can approximate the variance $V(lnY_t)$ of these lnY_{it} and, using equation (10), after having estimated this variance for each time period *t*, we are able to estimate the variance we require, namely, $V(lnX)$.

On the basis of the observations on lnY_t , as outlined above, we can also estimate, for each period *t,* the expectation, $E(\ln Y_t)$.

Using equation (13), we then also obtain that

$$E(\ln X) = E(\frac{1}{T}\sum_{t=1}^{T} \ln Y_t) = \frac{1}{T}\sum_{t=1}^{T} E(\ln Y_t). \quad (15)$$

We estimate $E(lnY_t)$ by $[\left(\frac{1}{n}\right)\sum_{i=1}^{n} lnY_{it}]$, the sample mean of the log observations for the t^{th} period. The mean of these sample means is then the estimate of $E(lnX)$, so that we not only estimate $V(lnX)$ but also $E(lnX)$.

We may now use the Taylor's expansion of lnX to obtain:

$$E(lnX) \approx lnE(X) - \frac{1}{2[E(X)]^2}V(X) \quad (16)$$

and

$$V(lnX) \approx \frac{1}{[E(X)]^2}V(X). \quad (17)$$

Combining equations (16) and (17), we obtain:

$$\frac{1}{2}V(lnX) + E(lnX) \approx \frac{1}{2[E(X)]^2}V(X) + E(lnX) \approx lnE(X). \quad (18)$$

Because we previously estimated both $V(lnX)$ and $E(lnX)$, we have now estimated $lnE(X)$.

We then derive also that

$$E(X) = e^{lnE(X)} \quad (19)$$

From equation (19), we derive also $[E(X)]^2$.

Using equation (20), we conclude that

$$V(X) \approx V(lnX)[E(X)]^2 \quad (20)$$

which enables us to estimate $V(X)$.

From equation (10), and using the well-known formula for the variance of the product of two uncorrelated random variables, we then obtain that:

$$V(Y_t) = V(X) + V(X\varepsilon_t)$$
$$= V(X) + \{[V(X)V(\varepsilon_t)] + [V(X)[E(\varepsilon_t)]^2] + [V(\varepsilon_t)[E(X)]^2]\}.$$

Because $E(\varepsilon_t) = 0$, the third term on the right-hand side is zero, and so we obtain that

$$V(\varepsilon_t) = \frac{V(Y_t) - V(X)}{V(X) + [E(X)]^2}. \quad (21)$$

Because we previously estimated the values of $V(Y_t)$, $V(X)$ and $[E(X)]^2$, using equation (21), we are able to estimate $V(\varepsilon_t) = \sigma_R^2$.

This allows then, using equation (9), to estimate the adjusted poverty line in the case of vulnerability and constant relative risk aversion.

5. EMPIRICAL ILLUSTRATIONS

Implementing either the additive or the multiplicative case requires estimating (or approximating) the value of the risk aversion parameter. In the additive case, the parameter value depends on the unit of measurement of the income, consumption or other well-being variable. To the best of our knowledge, no prior estimates of this parameter have been obtained using consumption or income in 2005 PPPs. In this paper, we implement the framework with multiplicative risks and present estimates for the case of constant relative risk aversion.

There have been numerous attempts to estimate the coefficient of relative risk aversion. Hartley et al. (2013) start their study by reviewing the literature on this topic, and mention, among the many papers they cite, the following results. Szpiro (1986) derived his estimates of the coefficient of relative risk aversion (CRRA) from time-series data on insurance premiums and concluded that the CRRA was close to 2. Barsky et al. (1997) worked with the US Health and Retirement Survey and estimated the CRRA to have a mean of about 12. Hersch and McDougall (1997) used data from the *Illinois Instant Riches* television game show and found evidence of a high value for the CRRA, up to 15. Jianakoplos and Bernasek (1998) analyzed US household portfolio data on risky assets and concluded that single women are more risk averse than single men because the former had a CRRA of 9 and 6 for the latter. Beetsma and Schotman (2001) used a Dutch game called *Lingo* and derived a range of 3 to 7 for the CRRA. Attanasio, Banks and Tanner (2002) used a large UK sample survey and obtained an estimate of the CRRA of 1.44. Chetty (2003) derived estimates of the CRRA on the basis of labor supply elasticities and found that the CRRA was close to 1. Fullenkamp, Tenorio and Battalio (2003) took the *Hoosier Millionaire* television game show as the data base and found that the CRRA varied between 0.64 and 1.76. Chiappori and Paiella (2011) preferred to use panel data because these data allow one to disentangle the impact of the shape of individual preferences and that of the correlation between preferences and wealth. They found that the median of the CRRA was around 2 but, for one fourth of the population, the CRRA was larger than 3. Gandelman and Hernández-Murillo (2011) used information on self-reports of subjective personal well-being from three datasets: the Gallup World Poll, the European Social Survey and the World Values Survey. They concluded that the CRRA varied between 0.79 and 1.44. Hartley, Lanot and Walker (2013) themselves analyzed data of the famous game show *Who Wants to be a Millionaire* and reached the conclusion that the CRRA was close to 1.

The short survey above on the CRRA shows clearly that there is quite a wide range of possible values for this coefficient, more or less from 0.5 to 15. The empirical illustration below covers mainly poor countries in Asia and it is reasonable to apply medium CRRA values to generate poverty lines. We therefore assume that the CRRA is equal to 3. More precisely, starting from an original poverty line of $1.25 (the official poverty line derived by the World Bank) which serves as a benchmark, we estimate what a vulnerability-adjusted poverty line would be for each country examined. The computations, presented in Tables 1 to 3, as well as those given in Appendix 2, are based on a technique originally proposed by Shorrocks and Wan (2009), which allows one to considerably increase the number of observations, even when starting from, say, only 10 observations for a given country and year (e.g., income deciles). This technique, known as “ungrouping income distributions”, is described in Appendix 1.

In Appendix 2, we present additional results where the CRRA is equal to 1.8, 5 and 10. Assuming that the CRRA is equal to 3, Table 1 shows that, for 2005, large values for vulnerability-adjusted poverty lines are observed for the People's Republic of China (PRC) ($1.88), Thailand ($1.56), Turkmenistan ($1.56), Georgia ($1.51), Malaysia ($1.51), and Viet Nam ($1.50). In 2010, the order did not change much—countries with high poverty lines include the PRC ($2.26), Malaysia ($1.82), Azerbaijan ($1.66), Viet Nam ($1.60), Thailand ($1.59), Tajikistan (1.58), and Turkmenistan ($1.56).

Table 1: Vulnerability-adjusted Poverty Lines for Countries in Asia and the Pacific (CRRA = 3)

Sub-region/Country	2005	2008	2010
Central and West Asia			
Armenia	1.39	1.45	1.39
Azerbaijan	1.46	1.60	1.66
Georgia	1.51	1.53	1.51
Kazakhstan	1.38	1.41	1.42
Kyrgyz Republic	1.36	1.56	1.49
Pakistan	1.40	1.39	1.47
Tajikistan	1.46	1.57	1.58
Turkmenistan	1.56	1.56	1.56
East Asia (PRC)	1.88	2.15	2.26
South Asia			
Bangladesh	1.35	1.37	1.38
Bhutan	1.36	1.44	1.50
India	1.37	1.39	1.40
Maldives	1.47	1.38	1.46
Nepal	1.43	1.50	1.56
Sri Lanka	1.42	1.45	1.45
Southeast Asia			
Cambodia	1.37	1.43	1.46
Indonesia	1.44	1.43	1.49
Lao PDR	1.38	1.41	1.47
Malaysia	1.51	1.81	1.82
Philippines	1.48	1.48	1.49
Thailand	1.56	1.55	1.59
Viet Nam	1.50	1.56	1.60
Pacific			
Fiji	1.41	1.46	1.48
Federated States of Micronesia (Urban)	1.38	1.40	1.41
Papua New Guinea	1.38	1.40	1.41
Timor-Leste	1.35	1.35	1.34

CRRA = coefficient of constant relative risk aversion.

Using these vulnerability-adjusted poverty lines, we computed the poverty rates and the number of poor (see Table 2). In the PRC, for example, we observe that once vulnerability is incorporated, its poverty rate becomes equal to 31.8% in 2005 and 28.7% in 2010. The corresponding rates are respectively: 30.6% in 2005 and 24.5% in 2010 for Pakistan; 56.4% in 2005 and 50.9% in 2010 for Bangladesh; 48.1% in 2005 and 41.6% in 2010 for India; 54.4% in 2005 and 39.6% in 2010 for Nepal; 29.6% in 2005 and 27.1% in 2010 for Indonesia; 30.0% in 2005 and 26.4% in 2010 for the Philippines; and 35.1% in 2005 and 25.4% in 2010 for Viet Nam.

Table 2: Poverty in Countries of Asia and the Pacific under Vulnerability-adjusted Poverty Lines (CRRA = 3)

	Poverty Rate (%)			**Number of Poor** (million)		
Sub-region/Country	**2005**	**2008**	**2010**	**2005**	**2008**	**2010**
Central and West Asia	**25.8**	**23.9**	**20.4**	**53.12**	**51.57**	**45.59**
Armenia	6.6	2.8	4.1	0.20	0.09	0.13
Azerbaijan	2.8	1.0	0.6	0.23	0.09	0.06
Georgia	21.7	21.1	23.7	0.95	0.93	1.05
Kazakhstan	1.4	0.1	0.5	0.21	0.02	0.07
Kyrgyz Republic	26.5	12.2	12.1	1.36	0.64	0.65
Pakistan	30.6	29.0	24.5	48.48	48.57	42.48
Tajikistan	25.0	18.3	16.4	1.61	1.22	1.13
Turkmenistan	1.6	0.4	0.2	0.08	0.02	0.01
East Asia (PRC)	**31.8**	**30.3**	**28.7**	**414.39**	**401.53**	**384.05**
South Asia	**48.6**	**46.0**	**42.0**	**646.82**	**637.38**	**599.28**
Bangladesh	56.4	53.6	50.9	79.24	77.92	75.62
Bhutan	22.8	15.2	9.1	0.15	0.11	0.07
India	48.1	45.6	41.6	549.20	543.56	509.96
Maldives	4.2	0.6	0.9	0.01	0.00	0.00
Nepal	54.4	46.5	39.6	14.83	13.43	11.87
Sri Lanka	17.0	11.6	8.5	3.38	2.37	1.77
Southeast Asia	**26.0**	**24.4**	**22.0**	**131.93**	**128.36**	**118.54**
Cambodia	39.4	30.8	23.3	5.27	4.25	3.29
Indonesia	29.6	30.9	27.1	67.18	72.49	64.94
Lao PDR	47.2	42.0	36.5	2.71	2.53	2.26
Malaysia	0.9	1.2	1.2	0.23	0.32	0.35
Philippines	30.0	27.0	26.4	25.68	24.30	24.63
Thailand	2.9	1.4	1.5	1.91	0.96	1.01
Viet Nam	35.1	27.6	25.4	28.95	23.51	22.06
Pacific	**47.6**	**43.0**	**40.5**	**3.79**	**3.65**	**3.59**
Fiji	21.6	9.2	12.3	0.18	0.08	0.11
Federated States of Micronesia (Urban)	33.2	35.1	35.2	0.01	0.01	0.01
Papua New Guinea	51.1	47.8	44.1	3.12	3.13	3.02
Timor-Leste	47.9	40.5	40.3	0.48	0.44	0.45
Developing Asia	**37.3**	**35.3**	**32.6**	**1,250.04**	**1,222.50**	**1,151.05**

Note: The data in columns 2 to 4 indicate the value of the headcount ratios when the original poverty line of $1.25 is adjusted for vulnerability with a coefficient of constant relative risk aversion (CRRA) equal to 3. The data in columns 5 to 7 give the corresponding numbers of poor.

The term “Developing Asia” covers all the countries that appear in the table.

In Table 3, we present a summary of the results concerning the headcount ratios and the number of poor for developing Asia as a whole in 2005, 2008 and 2010, under various assumptions regarding the coefficient of relative risk aversion. We also show what the headcount ratio and the number of poor are, when no adjustment is made for vulnerability so that the poverty line is assumed to be equal to $1.25 (our benchmark). We observe the important increases in the headcount ratio when vulnerability is taken into account, even when the CRRA is equal to 1.2. Note also that the headcount ratio (and the number of poor) increases with the CRRA but only up to a value of 5. When the CRRA is equal to 10, the values of the headcount ratio (and the number of poor) are smaller than when the CRRA is equal to 5. Assuming a coefficient of relative of risk aversion equal to 3, we can see that in 2005 there are 350,000 more poor than under the benchmark case (poverty line equal to $1.25). In 2010 the difference is even higher (more than 400,000 additional poor).

Table 3: Vulnerability-adjusted Headcount Ratios and Number of Poor for Developing Asia: Summary of Results

	Headcount Ratios (2005)	Headcount Ratios (2008)	Headcount Ratios (2010)	Number of Poor (2005)	Number of Poor (2008)	Number of Poor (2010)
Benchmark (poverty line = $1.25)	26.9	23.9	20.7	901.96	827.57	733.06
CRRA=1.2	32.1	30.2	27.5	1,077.82	1,046.17	973.39
CRRA=1.5	33.3	31.6	28.9	1,118.57	1,093.67	1,021.42
CRRA=1.8	34.4	32.7	30.0	1,154.45	1,132.99	1,061.90
CRRA=2	35.0	33.4	30.7	1,175.12	1,154.82	1,084.06
CRRA=3	**37.3**	**35.3**	**32.6**	**1,250.04**	**1,222.50**	**1,151.05**
CRRA=5	38.9	36.5	33.6	1,306.34	1,262.14	1,186.69
CRRA=10	38.5	35.5	32.4	1,293.27	1,229.06	1,146.67

If we now take a look at specific countries (see Table A.1), for example, for the year 2010, we observe that under the benchmark of a poverty line of $1.25, the headcount ratio is equal to 11.6% in the PRC, 32.7% in India, 18.2% in Indonesia, 13.5% in Pakistan and 18.4% in the Philippines. When we adjust the poverty line for vulnerability and select a CRRA of 1.8, the corresponding percentages are 23.2 (PRC), 36.7 (India), 21.8 (Indonesia), 18.8 (Pakistan) and 21.8 (the Philippines). With a CRRA equal to 5 the headcount ratios in these countries become 26.7% (PRC), 44.9% (India), 29.5% (Indonesia), 27.9% (Pakistan) and 26.4% (the Philippines). Finally, when the CRRA is equal to 10 the headcount ratios are 22.0% in PRC, 46.6% in India, 29.7% in Indonesia, 28.4% in Pakistan, and 28.6% in the Philippines.

We thus observe more than a doubling of the headcount ratio in Pakistan and PRC when we compare the situation under a poverty line of $1.25 with that adjusted for vulnerability with a CRRA equal to 5. For India the increase is higher than 35%, for the Philippines it is higher than 50% and for Indonesia the rise is of 60%. Therefore, when vulnerability is taken into account, the extent of poverty in the most populated countries of Asia, and hence in Asia as a whole is modified significantly.

6. CONCLUSIONS

In this paper, we have addressed the problem of modifying the poverty line when the income distribution is affected by vulnerability. The formal framework considered in the paper relies on the Ligon and Schechter (2003) definition of vulnerability as expected utility loss. Under alternative assumptions about the uncertainty (noise) that indicates vulnerability, it is shown that for the constant absolute or relative Arrow-Pratt risk aversions, the tailored poverty line becomes either an absolute or relative shift of the current poverty line. The empirical illustration, based on data from various Asian and Pacific countries, assumed constant relative risk aversion and showed the important impact of vulnerability on the number of poor in various Asian countries.

We thus observed generally important increases in the headcount ratio when vulnerability is taken into account. For example, assuming a coefficient of relative of risk aversion equal to 3, there were in 2005 350,000 more poor than under the benchmark case (poverty line equal to $1.25). In 2010, the difference was even higher (more than 400,000 additional poor). Looking at specific countries, we observed more than a doubling of the headcount ratio in Pakistan and the PRC when we compared the situation under a poverty line of $1.25 with that adjusted for vulnerability with a CRRA equal to 5. For India, the Philippines and Indonesia, the increases were respectively higher than 35%, 50% and 60%. It is thus clear that when vulnerability is taken into account, the extent of poverty in the most populated countries of Asia, and, hence, in Asia as a whole, is modified significantly.

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APPENDIX 1: ON SHORROCKS AND WAN'S (2009) "UNGROUPING INCOME DISTRIBUTIONS"

Assume a Lorenz curve with $(m+1)$ coordinates (p_k^*, L_k^*) where p_k^* and L_k^* $(k = 1, \dots, m)$ refer respectively to the cumulative shares in the total population and in total income of income classes 1 to *k*, while $p_0^* = L_0^* = 0$. These Lorenz coordinates can, for example, refer to decile shares published on a given country. Because the corresponding average income is often not available, it is assumed to be equal to 1 so that the mean income μ_k^* of class *k* is expressed as

$$\mu_k^* = \frac{L_k^* - L_{k-1}^*}{p_k^* - p_{k-1}^*}, k = 1, \dots, m. \quad \text{(A-1)}$$

The goal is to obtain a synthetic sample of *n* equally weighted observations whose mean value is 1 and which conform to the original data. These *n* observations are therefore partitioned into *m* non-overlapping and ordered groups having each $m_k = n(p_k^* - p_{k-1}^*)$ observations. Call x_{ki} the i^{th} observation in class *k*, the sample mean of this class being μ_k.

The algorithm proposed by Shorrocks and Wan (2009) includes two stages. The first consists of building an initial sample with unit mean which is generated from a parametric form fitted to the grouped data (see, for example, Ryu and Slottje (1999) for a survey of various parametrizations of the Lorenz curve[5]).

In the second stage, the algorithm adjusts the observations generated in the initial sample to the true values available from the grouped data. More precisely, the initial sample value, x_j, assumed to belong to class *k*, is transformed into an intermediate value $\widehat{x_j}$ via the following rule:

$$\frac{\widehat{x_j} - \mu_k^*}{\mu_{k+1}^* - \mu_k^*} = \frac{x_j - \mu_k}{\mu_{k+1} - \mu_k} \quad \text{(A-2)}$$

For the first class, we write that

$$\frac{\widehat{x_j}}{\mu_1^*} = \frac{x_j}{\mu_1} \quad \text{for } x_j \leq \mu_1 \quad \text{(A-3)}$$

while, for the last class, we have

$$\frac{\widehat{x_j}}{\mu_m^*} = \frac{x_j}{\mu_m} \quad \text{for } x_j \geq \mu_m. \quad \text{(A-4)}$$

Obviously, in the next iteration, the intermediate values $\widehat{x_j}$ are themselves transformed into new values until the algorithm produces an ordered sample that exactly replicates the properties of the original grouped data. Convergence is, in fact, very quickly obtained.

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5 Shorrocks and Wan chose to generate the initial sample on the basis of a lognormal distribution. For more details, see, Shorrocks and Wan (2009).

APPENDIX 2: VULNERABILITY-ADJUSTED HEADCOUNT RATIOS AND NUMBERS OF POOR: DETAILED RESULTS BY COUNTRY

Table A.1: The Benchmark Case (without Vulnerability Adjustment): Headcount Ratios and Number of Poor when the Poverty Line is $1.25

Sub-region/Country	Headcount Ratio (%) 2005	Headcount Ratio (%) 2008	Headcount Ratio (%) 2010	Number of Poor (million) 2005	Number of Poor (million) 2008	Number of Poor (million) 2010
Central and West Asia	18.8	17.1	11.2	38.79	37.05	25.14
Armenia	4.0	1.3	2.5	0.12	0.04	0.08
Azerbaijan	1.5	0.4	0.3	0.13	0.04	0.03
Georgia	16.0	15.3	18.0	0.70	0.67	0.80
Kazakhstan	0.8	0.1	0.3	0.12	0.02	0.04
Kyrgyz Republic	22.9	6.4	6.7	1.18	0.34	0.36
Pakistan	22.3	21.0	13.5	35.38	35.23	23.38
Tajikistan	17.7	10.7	6.6	1.14	0.72	0.45
Turkmenistan	0.5	0.1	0.1	0.02	0.01	0.00
East Asia (PRC)	16.3	13.1	11.6	211.85	173.00	155.51
South Asia	41.5	37.8	33.2	552.03	523.85	472.72
Bangladesh	50.5	46.6	43.3	70.96	67.82	64.31
Bhutan	18.9	9.3	4.4	0.12	0.07	0.03
India	40.8	37.4	32.7	466.30	445.02	400.08
Maldives	2.3	0.2	0.2	0.01	0.00	0.00
Nepal	46.3	33.9	24.8	12.64	9.80	7.44
Sri Lanka	10.1	5.6	4.1	2.00	1.14	0.86
Southeast Asia	18.9	17.2	14.2	95.87	90.47	76.59
Cambodia	33.8	22.8	14.7	4.51	3.14	2.08
Indonesia	21.4	22.6	18.2	48.73	53.19	43.32
Lao PDR	39.5	33.9	26.0	2.27	2.04	1.61
Malaysia	0.4	–	–	0.11	–	–
Philippines	22.2	19.4	18.4	19.02	17.49	17.18
Thailand	1.0	0.4	0.4	0.68	0.25	0.26
Viet Nam	24.9	16.9	14.0	20.55	14.34	12.14
Pacific	43.0	37.8	34.9	3.42	3.21	3.10
Fiji	17.9	5.0	6.1	0.15	0.04	0.05
Federated States of Micronesia (Urban)	30.6	32.1	32.0	0.01	0.01	0.01
Papua New Guinea	46.6	42.5	38.6	2.84	2.78	2.65
Timor-Leste	42.0	34.7	34.7	0.42	0.37	0.39
Developing Asia	26.9	23.9	20.7	901.96	827.57	733.06

Note: The term “Developing Asia” covers all the countries that appear in the table.

Table A.2: The Coefficient of Relative Risk Aversion (CRRA) is Equal to 1.8

Sub-region/Country	**Headcount Ratio** (%) **2005**	**Headcount Ratio** (%) **2008**	**Headcount Ratio** (%) **2010**	**Number of Poor** (million) **2005**	**Number of Poor** (million) **2008**	**Number of Poor** (million) **2010**
Central and West Asia	21.8	20.0	15.6	44.98	43.29	35.01
Armenia	5.0	1.9	3.1	0.15	0.06	0.10
Azerbaijan	2.0	0.6	0.4	0.17	0.05	0.04
Georgia	18.8	18.2	20.4	0.82	0.80	0.91
Kazakhstan	1.0	0.1	0.3	0.15	0.02	0.05
Kyrgyz Republic	24.5	8.9	9.0	1.26	0.47	0.48
Pakistan	25.9	24.5	18.8	41.05	40.97	32.67
Tajikistan	20.6	13.7	10.9	1.33	0.92	0.75
Turkmenistan	0.9	0.2	0.1	0.04	0.01	0.01
East Asia (PRC)	24.9	24.1	23.2	324.12	318.93	310.71
South Asia	44.6	41.3	37.1	593.72	573.33	529.30
Bangladesh	53.0	49.7	46.6	74.56	72.25	69.31
Bhutan	20.6	11.9	5.9	0.14	0.08	0.04
India	44.0	41.0	36.7	502.74	487.99	449.30
Maldives	3.1	0.4	0.5	0.01	0.00	0.00
Nepal	50.1	39.8	31.9	13.66	11.50	9.55
Sri Lanka	13.2	7.3	5.3	2.62	1.50	1.10
Southeast Asia	22.0	20.4	17.7	111.43	107.23	95.05
Cambodia	36.3	26.4	18.8	4.84	3.64	2.66
Indonesia	24.7	26.4	21.8	56.24	61.92	52.34
Lao PDR	42.9	37.5	30.8	2.47	2.26	1.91
Malaysia	0.6	0.5	0.5	0.15	0.13	0.15
Philippines	25.9	22.5	21.8	22.15	20.33	20.33
Thailand	1.8	0.8	0.8	1.19	0.53	0.55
Viet Nam	29.6	21.6	19.7	24.39	18.42	17.09
Pacific	45.0	40.0	37.4	3.58	3.40	3.32
Fiji	19.5	6.2	8.6	0.16	0.05	0.07
Federated States of Micronesia (Urban)	31.8	33.4	33.4	0.01	0.01	0.01
Papua New Guinea	48.6	44.9	41.1	2.96	2.94	2.82
Timor-Leste	44.5	37.2	37.1	0.45	0.40	0.42
Developing Asia	32.1	30.2	27.5	1,077.82	1,046.17	973.39

Note: The term "Developing Asia" covers all the countries that appear in the table.

Table A.3: The Coefficient of Relative Risk Aversion (CRRA) is Equal to 5

Sub-region/Country	**Headcount Ratio** (%) **2005**	**Headcount Ratio** (%) **2008**	**Headcount Ratio** (%) **2010**	**Number of Poor** (million) **2005**	**Number of Poor** (million) **2008**	**Number of Poor** (million) **2010**
Central and West Asia	28.4	26.4	23.2	58.57	57.14	51.87
Armenia	7.8	3.5	5.0	0.24	0.11	0.16
Azerbaijan	3.3	1.1	0.7	0.27	0.10	0.06
Georgia	23.3	22.6	25.3	1.02	0.99	1.13
Kazakhstan	1.7	0.1	0.6	0.26	0.02	0.09
Kyrgyz Republic	28.1	13.6	13.8	1.44	0.72	0.74
Pakistan	33.7	32.2	27.9	53.48	53.84	48.42
Tajikistan	27.5	20.1	18.2	1.77	1.34	1.25
Turkmenistan	1.9	0.5	0.3	0.09	0.03	0.01
East Asia (PRC)	32.0	28.9	26.7	417.13	382.31	357.09
South Asia	51.4	49.0	45.3	683.74	679.25	644.90
Bangladesh	58.8	56.4	53.8	82.72	81.98	79.98
Bhutan	24.4	17.0	10.9	0.16	0.12	0.08
India	50.9	48.7	44.9	581.57	579.98	549.84
Maldives	4.9	0.8	1.2	0.01	0.00	0.00
Nepal	56.8	49.6	42.5	15.49	14.33	12.73
Sri Lanka	19.1	13.9	10.8	3.80	2.84	2.26
Southeast Asia	28.2	26.6	24.0	142.97	139.62	129.08
Cambodia	41.6	33.5	26.0	5.56	4.63	3.68
Indonesia	32.4	33.6	29.5	73.57	79.06	70.85
Lao PDR	50.2	44.8	39.4	2.88	2.70	2.44
Malaysia	1.1	1.2	1.3	0.27	0.34	0.37
Philippines	32.0	29.1	28.6	27.35	26.25	26.64
Thailand	3.4	1.7	1.7	2.24	1.18	1.20
Viet Nam	37.7	29.9	27.5	31.10	25.46	23.89
Pacific	49.4	44.9	42.3	3.93	3.81	3.76
Fiji	23.0	11.1	14.1	0.19	0.09	0.12
Federated States of Micronesia (Urban)	34.2	36.3	36.3	0.01	0.01	0.01
Papua New Guinea	52.8	49.5	45.8	3.22	3.25	3.14
Timor-Leste	50.6	43.1	42.8	0.51	0.47	0.48
Developing Asia	38.9	36.5	33.6	1,306.34	1,262.14	1,186.69

Note: The term “Developing Asia” covers all the countries that appear in the table.

Table A.4: The Coefficient of Relative Risk Aversion (CRRA) is Equal to 10

Sub-region/Country	Headcount Ratio (%) 2005	Headcount Ratio (%) 2008	Headcount Ratio (%) 2010	Number of Poor (million) 2005	Number of Poor (million) 2008	Number of Poor (million) 2010
Central and West Asia	29.8	27.9	23.5	61.34	60.41	52.68
Armenia	8.6	3.6	5.8	0.26	0.11	0.18
Azerbaijan	3.4	1.0	0.6	0.28	0.09	0.06
Georgia	23.2	22.4	25.2	1.01	0.98	1.12
Kazakhstan	1.9	0.2	0.6	0.29	0.02	0.10
Kyrgyz Republic	29.3	13.1	13.8	1.51	0.69	0.74
Pakistan	35.4	34.2	28.4	56.09	57.18	49.28
Tajikistan	28.0	19.5	17.3	1.81	1.31	1.19
Turkmenistan	1.8	0.5	0.3	0.08	0.02	0.01
East Asia (PRC)	28.5	24.1	22.0	371.61	319.71	294.00
South Asia	53.4	50.7	46.9	710.77	703.43	667.90
Bangladesh	60.9	58.4	55.7	85.62	84.95	82.87
Bhutan	25.6	17.5	10.9	0.17	0.12	0.08
India	53.0	50.5	46.6	605.26	601.07	570.17
Maldives	5.1	1.0	1.3	0.02	0.00	0.00
Nepal	57.6	49.5	41.4	15.72	14.32	12.41
Sri Lanka	20.1	14.5	11.4	3.98	2.97	2.37
Southeast Asia	28.7	26.9	23.8	145.53	141.61	128.25
Cambodia	43.3	34.5	26.5	5.78	4.77	3.75
Indonesia	33.3	34.7	29.7	75.74	81.50	71.22
Lao PDR	52.0	46.1	39.7	2.99	2.78	2.46
Malaysia	1.0	0.9	1.0	0.27	0.25	0.27
Philippines	32.2	29.3	28.6	27.53	26.43	26.68
Thailand	3.2	1.6	1.6	2.12	1.12	1.09
Viet Nam	37.7	29.1	26.2	31.10	24.77	22.78
Pacific	50.6	45.9	43.3	4.03	3.90	3.84
Fiji	23.6	11.6	14.4	0.19	0.10	0.12
Federated States of Micronesia (Urban)	34.9	36.8	36.8	0.01	0.01	0.01
Papua New Guinea	53.9	50.4	46.6	3.29	3.30	3.20
Timor-Leste	53.0	45.5	45.2	0.53	0.49	0.51
Developing Asia	38.5	35.5	32.4	1,293.27	1,229.06	1,146.67

Note: The term “Developing Asia” covers all the countries that appear in the table.